Bandwidth Allocation in Radio Grid Networks

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In this paper we give exact or almost exact bounds for the continuous gathering problem on grids. Under very general hypothesis on the traffic demand, we mainly prove that the throughput is determined by the bottleneck around the base station. We deal with two cases: the base station located in the center and in the corner. We use dual lower bounds and describe a protocol which is optimal when the traffic is uniform.

1 Introduction

The routing problem of steady traffic demands in a radio network has been studied extensively in the literature. In [KMP08] it was proven that if traffic demands are sufficiently steady the problem can be expressed in an independent form of the interference model as the *Round Weighting Problem (RWP)*. We deal with a special case of *RWP* that considers the gathering of the flow. We represent nodes by the vertices of a transmission graph $G=(V,E)$. The edges of this graph connect nodes that can effectively communicate. The interference model is introduced by providing an implicit definition of the set of possible *rounds* $\mathcal{R}$. We take a simple model of interference where a round is any set of pairwise disjoint edges at distance at least d_I^s+1 (Manhattan distance). It defines a symmetric interference model that permits the calls to happen in both directions (download or upload).

The *RWP* is then defined as follows: The traffic demands are represented by a flow demand $f(u,v): V\times V\rightarrow\mathbb{N}$ and one wishes to find a (positive) weight function $w:\mathcal{R}\rightarrow\mathbb{R}^+$ *that enables the flow demands to be carried over the network*. The objective MinRW is then to minimize the total weight (namely $w(\mathcal{R})=\sum_{R\in\mathcal{R}}w(R)$).

In the case of a general transmission graph with an arbitrary traffic pattern the problem is very difficult to approximate, indeed, to approximate the *RWP* within $n^{1-\varepsilon}$ is NP-Hard [KMP08]. A practically important case is the *Gathering* (or personalized broadcasting): the traffic pattern corresponds then to a simple flow, i.e. all demands are directed toward a single node called the BS (Base Station). Gathering is easier to approximate since a simple 4–approximation does exist, but the problem remains NP-Hard. Instances on a grid are tractable mainly due to the local structure of the grid. Note also that the structure of the transmission graph plays a central role, if G is a grid or a unit disk graph the *RWP* admits a PTAS but remains NP-Hard.

In [BP05], a similar problem, the *Round Scheduling Problem (RSP)* was treated. The relation with the *RWP* is the following: if one must repeat rounds scheduling many times then the problem

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is equivalent to the *RWP* . The *RSP* is quite harder to solve than our problem which can be be considered either as a limited case or relaxation. Not surprisingly we obtain not only simpler formulae than Bermond and Peters, but they are valid for a larger class of traffic patterns. Note that, in [BP05] $d_I > 1$ and it is not symmetric because they deal with the exact case of gathering (directed interference). In this paper we study *RWP* on 2-dimensional grid graphs considering the interference distance $d_I^s = 1$ (but our method can be extended to any d_I^s). The most basic case is $d_I^s = 0$, where $\mathcal{R}$ is simply the set of the matchings of G. Since all the traffic demands have as destination/source the BS ($f(u,v) = 0$ when $v \neq \mathsf{BS}$) we simply note $f(u,\mathsf{BS})$ as $f(u)$. We denote S_j the set of nodes at distance j of the BS, and $E_j = (S_j, S_{j-1})$ the set of arcs of the grid connecting the nodes in S_j to S_{j-1}. For example, S_1 represents the set of 4 nodes neighbors of BS and E_1 consists of the 4 edges ending in BS.

2 Lower bounds

First we recall the dual lower bound from [KMP08]: Given a (positive) *length* function $l(e)$ on the edges of the transmission graph, we define $l(u,\mathsf{BS})$ or simply $l(u)$ as the minimum *length* of a path connecting u to BS. Since the problem is homogeneous the dual is indeed: **Maximize**$\{\sum l(u)f(u)\}$ with $l(R) \leq 1, \forall R \in \mathcal{R}$. Then $MinRW \geq \frac{\sum l(u)f(u)}{max_{R\in\mathcal{R}} l(R)}$.

3 Uniform traffic case

In the uniform case, we consider that all the nodes have the same flow demand. Without loss of generality, we suppose $f(u) = 1$ for all $u \in V$ except BS. We denote T the total traffic demand $\sum_{v\in V} f(u)$. In this case, $T = N - 1$ where N is the number of nodes of the grid. We define T' as the traffic demand that must cross the arcs at distance 2 of BS in order to be gathered. Thus, $T' = T - \sum_{u\in\Gamma(\mathsf{BS})} f(u)$, where $\Gamma(\mathsf{BS})$ denotes the set of neighbors of BS.

3.1 Base Station in the center

Theorem 1 *Given a grid with the* BS *in the center,* $MinRW = T + \frac{T'}{4} = \frac{5}{4}N - \frac{9}{4}$.

Proof: The proof has two parts. The first part consists in finding a feasible solution for the dual problem. This solution is then, a lower bound for our *RWP* . In the second part, we find an upper bound given by a feasible solution of the original problem. Since there is no gap between both lower and upper bounds, the result follows.

(Lower Bound) The edges with strictly positive values are depicted in figure 1. The edges $e \in E_1$ have $l(e) = 1$. It comes from the fact that at most one edge in E_1 can be activated for each round. The edges $e \in E_2$ have $l(e) = \frac{1}{4}$, because it is possible to activate at most 4 edges in a round. For all the remaining edges $l(e) = 0$. The minimum *length* of the path connecting u to BS, $l(u)$, is therefore $\frac{5}{4}$ except for the 4 neighbors of BS.

(Upper Bound) We gather the traffic using the 4 adjacent arcs to the BS. Moreover, all the flow at distance larger than 2 will be collected by the 4 full arcs at distance 2 as shown in figures 1 and 2. We denote e_i^1 $(1 \leq i \leq 4)$ the 4 arcs adjacent to the BS and e_i^2 the 4 arcs at distance 2. We define $\mathcal{R}_i$ ($\mathcal{R}_1$, $\mathcal{R}_2$, $\mathcal{R}_3$ and $\mathcal{R}_4$) as the subset of rounds using the arc e_i^1 and $\mathcal{R}_5$ as the subset of rounds using e_i^2. Then, in order to attain the bound, we need to use only rounds in these subsets.

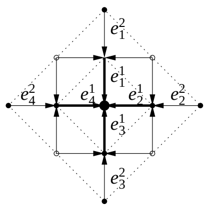

Fig. 1: Dual values

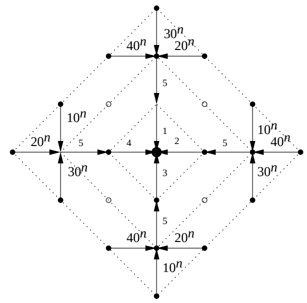

Fig. 2: *Normal* rounds R_{i0^n}

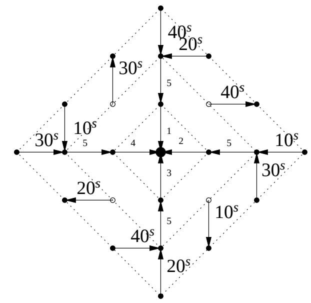

Fig. 3: *Special* rounds R_{i0^s}

Because of $d_I^s = 1$, to avoid interference, we will use different rounds for the calls in E_j, E_{j+1} and E_{j+2}. So, we have 13 types of rounds called $R_5 \in \mathcal{R}_5$ and $R_{ij} \in \mathcal{R}_i$, $1 \le i \le 4$, $0 \le j \le 2$. R_{ij} will contain arcs either e_i^1 or arcs in E_{j+3p} $(p \ge 1)$. R_5 contains only the arcs e_i^2. Note that, except E_2, each E_j use only rounds type $R_{i,j \bmod 3}$. We will choose the weights such that $w(R_{ij}) = \frac{T}{12}$ and $w(R_5) = \frac{T'}{4}$. Doing so we attain the lower bound but we have a problem with the 4 non-filled nodes in S_2 (in figure 3) which cannot be directly routed to BS via the edges in e_i^2. If we use an edge of E_2 but different from e_i^2, we can activate at the same round at most 3 edges of E_2 instead 4 (from e_i^2) and we will not reach the lower bound.

To deal with this difficulty, we split each round of type R_{i0} in two new rounds: The *special* round R_{i0^s} used to move the flow of one problematic node from S_2 to S_3 and the *normal* round R_{i0^n} where all the arcs are directed to the BS. The weights proposed for these rounds are $w(R_{i0^s}) = 1$, and $w(R_{i0^n}) = \frac{T}{12} - 1$. Note that, for arcs in E_j with $j \ge 3$, calls used by both types of rounds R_{i0^s} and R_{i0^n} are exactly the same. The differences between these types of rounds in arcs in E_1, E_2 and E_3 are presented in figures 2 and 3. Finally, we have used the rounds R_{i0^s}, R_{i0^n}, R_{i1}, R_{i2}, R_5, $1 \le i \le 4$. Their respective weights are $w(R_{i0^s}) = 1$, $w(R_{i0^n}) = \frac{T}{12} - 1$, $w(R_{ij}) = \frac{T}{12}$, and $w(R_5) = \frac{T'}{4}$.

Now, we need to show that there is a routing of the flow that respects the capacity induced of the arcs by our round weights. Note first that, globally, the capacity by each E_i, $i \ge 2$ is at least T'. This capacity is enough to transmit the flow desired between E_{i+1} to E_i and so on. Now, we propose a routing such that all the nodes in S_i receive the same quantity of flow from S_{i+1}. A special case occurs when distributing the flow between the nodes in S_3. As well as considering the flow from S_4 we need to consider the flow from the 4 special nodes in S_2. Note that by the symmetry of the grid, we can only take into account the routing of one quadrant of the grid, and then we repeat and rotate the configuration to the rest of the quadrants. ∎

3.2 Base Station in the corner

Theorem 2 *Given a grid with the* BS *in the corner,* $MinRW = T + \frac{T'}{2} + \frac{1}{2} = \frac{3}{2}N - 2$.

Proof: The structure of the proof is similar to Theorem 1.

(Lower Bound) The values of the *lengths* $l(e)$ are depicted in figure 4. The minimum *length* of the path connecting u to BS, $l(u)$, is therefore $\frac{3}{2}$ except for X and Y which is $l(u) = 1$, and for the node z which is $l(u) = 2$. Then $MinRW \ge T + \frac{T'}{2} + \frac{1}{2}$, where the term $\frac{1}{2}$ can be explained by the extra cost needed to send traffic from node z (the problematic node) to the usual route.

(Upper Bound) According to the dual values shown in figure 4, any scheme that costs about $\frac{3T}{2}$ must route $\sim \frac{T}{2}$ units of traffic through each of the nodes X and Y. We only need three

sets of rounds $\mathcal{R}_i$, each set must be split as shown in figure 5 to completely cover the grid. Let $T" = T' - f(z) = T' - 1$, we propose the respective weights: $w(R_1) = \frac{T}{2}$, $w(R_2) = \frac{T}{2}$, $w(R_{3A}) = \frac{T"}{2}$ and $w(R_{3B}) = 1$. The set R_3 will be used to cover the arcs E_i when $(i+1) \bmod 3 = 0$, we use R_1 and R_2 otherwise. It follows that we can cover the arcs E_i at least $T"$ times. Thus, we used a total weight of $T + \frac{T"}{2} + 1$ that matches the lower bound. A possible routing respecting our round weights (induced capacity) is such that all the nodes in S_i receive the same quantity of flow from S_{i+1}. ■

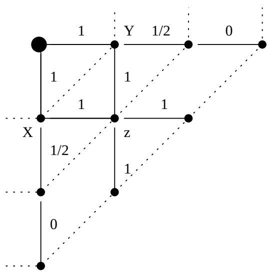

Fig. 4: Dual values

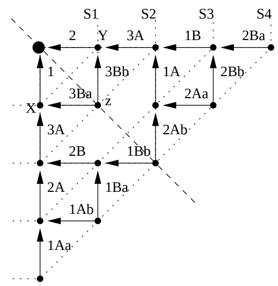

Fig. 5: Rounds Capacity

4 Generalizations

The proof given for an arbitrary traffic can be generalized to most traffics as follows. From lemma 1, we remark that outside the radius 3 ball the round R_5 is never used. So, a weight of $T/4$ is unused in most of the network and can be used to balance the traffic a priori. Thus, balancing the traffic turns into a transportation problem in the grid. We can prove that a non uniform demand can be re-routed to a uniform one if no node contains more than βT units of traffic, and the $5T/4$ bound holds. The same idea can also be applied to the case of an arbitrary BS location. The flow must then be routed toward the radius 3 ball in a uniform way and this is possible as soon as the BS is far enough (distance $\geq d_a$) from the borders. The quality of the constants β, d_a depend on how precise are the algorithms.

Another demonstration for the same results consists in finding k long non-interfering paths (with k the degree of BS) for each node through the BS. The gathering protocol is then quite simple since it sends the traffic of each node using the k paths in a balanced way. This should prove that the gathering time is $T(1 + \frac{1}{k}) + o(T)$ for most practical networks.

References

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